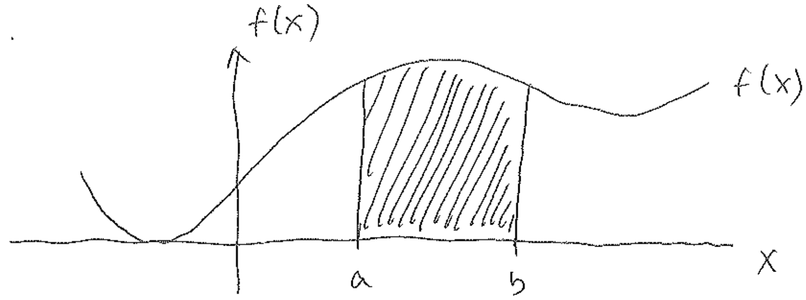
AMS 131 4/5/18

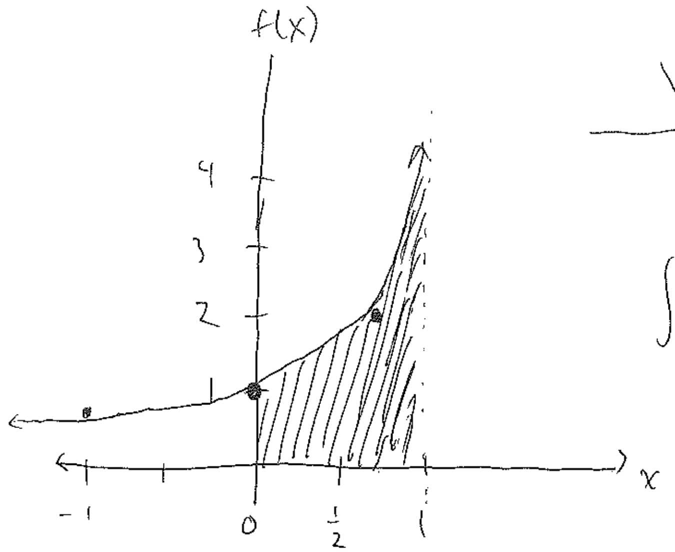
Review of Math Pre-requisites

- Appendix A of B & H
- Compute $\int_0^1 \frac{1}{\sqrt{1-x}}\,dx$

Q: What does this integral physically represent?

A: For $f(x) \geq 0$, $\int_a^b f(x)\,dx$ is equal to the area between $f(x)$ and the x-axis over the interval $x=a$ to $x=b$.





$$\int_0^1 \frac{1}{\sqrt{1-x}}\,dx = \int_0^1 (1-x)^{-\frac{1}{2}}\,dx$$

$$= -2(1-x)^{\frac{1}{2}} \Big|_0^1$$

$$= -2\left[(1-1)^{\frac{1}{2}} - (1-0)^{\frac{1}{2}}\right]$$

$$= -2(0-1) = +2$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$$

$$\frac{d}{dx}x^n = n x^{n-1}$$

other techniques:

substitution: $\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$, $\int_0^1 2x e^{-3x^2}\,dx$

integration by parts: $\int_0^1 2x e^{-3x}\,dx$

Double integrals

Compute $\int_0^1 \int_0^2 (x^2 y + x)\, dy\, dx$

$$= \int_0^1 \left(x^2 \frac{y^2}{2} + xy \right)\Bigg|_{y=0}^{y=2} dx$$

$$= \int_0^1 \left[(x^2 \cdot 2 + x \cdot 2) - (0+0) \right] dx$$

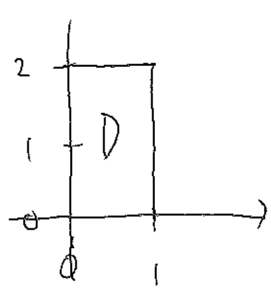
$$= \int_0^1 (2x^2 + 2x)\, dx$$

$$= \left(\frac{2}{3} x^3 + x^2 \right)\Bigg|_0^1 = \left(\frac{2}{3} + 1 \right) - (0+0) = \frac{5}{3}$$

Q: What does this integral represent?

For $f(x,y) \geq 0$, $\int_a^b \int_c^d f(x,y)\, dy\, dx$ is equal to the volume of the solid that lies above the rectangle $[a,b] \times [c,d]$ below the surface $f(x,y)$.

(In other words, the volume between $f(x,y)$ and the xy-plane over the rectangle $[a,b] \times [c,d]$).



D: base of the solid. (region of xy-plane)

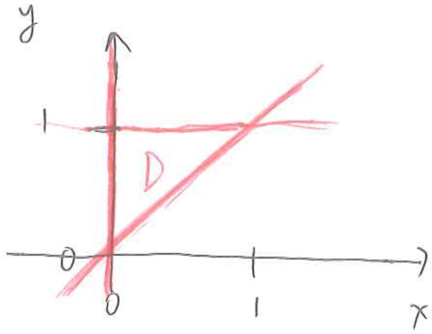
Q: What if this region is not rectangular?

Example: B&H

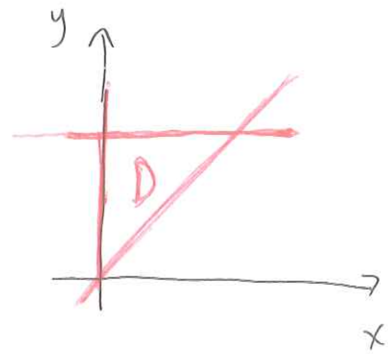
$$\int_0^1 \int_0^y (x-y)^2 \, dx \, dy = \frac{1}{12}$$

$$\int_0^1 \int_x^1 (x-y)^2 \, dy \, dx = \frac{1}{12}$$

What is the region D that is being integrated over?



$x = 0$ to $x = y$



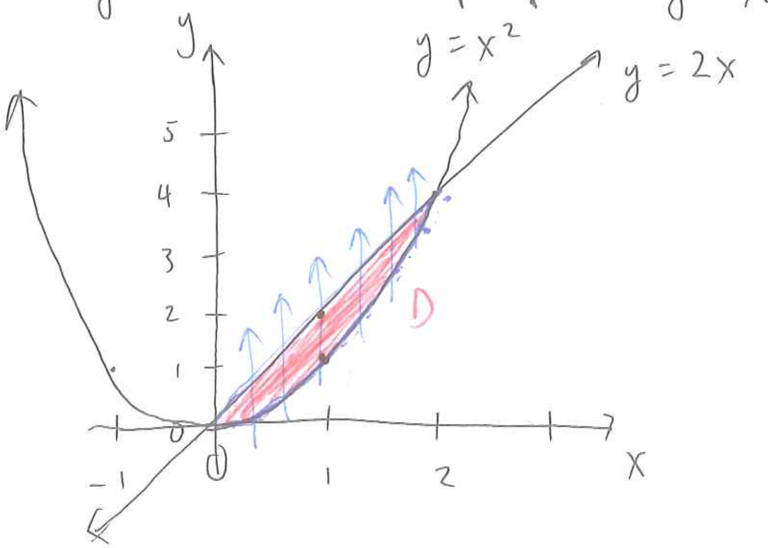
$y = x$ to $y = 1$

Notice that whenever the region is not rectangular, the limits of the inner integral contain/involve the outer variable of integration.

Also notice that the limits of the outer integral are numeric values and correspond to the range of the outer variable of integration.

Example 2:

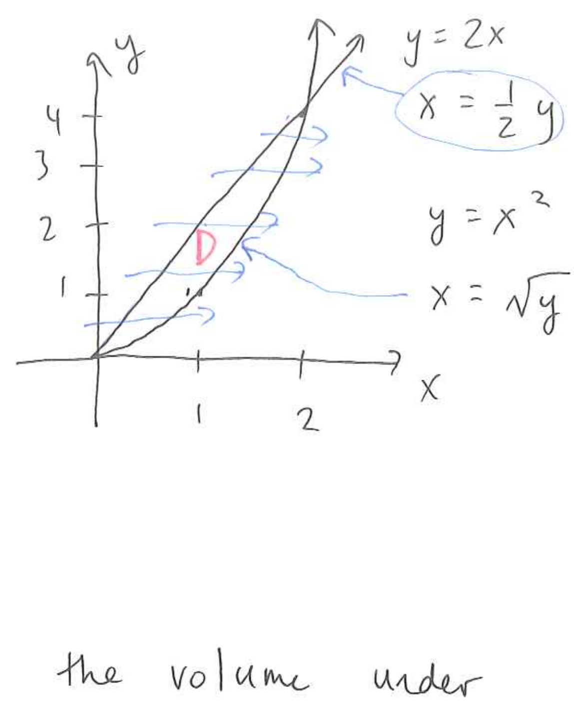
Find the volume of the solid that lies under $f(x,y) = x^2+y^2$ and above the region D in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.



$dA = dx\,dy$ or

$dA = dy\,dx$

$$\text{volume} = \iint_D (x^2+y^2)\,dA$$



x first:

$$\int_0^4 \int_{\frac{1}{2}y}^{\sqrt{y}} (x^2+y^2)\,dx\,dy = \frac{216}{35}$$

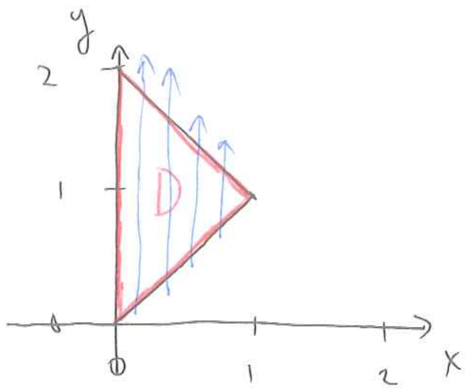
y first:

$$\int_0^2 \int_{x^2}^{2x} (x^2+y^2)\,dy\,dx = \frac{216}{35}$$

Example 3: (more practice) Find the volume under $f(x,y) = x^2 y$ above the region $D = \{(x,y) : x^2 \le y \le 1\}$.

Ans: $\frac{4}{21}$

Example 4 (one direction is easier) (x first or y first ?)



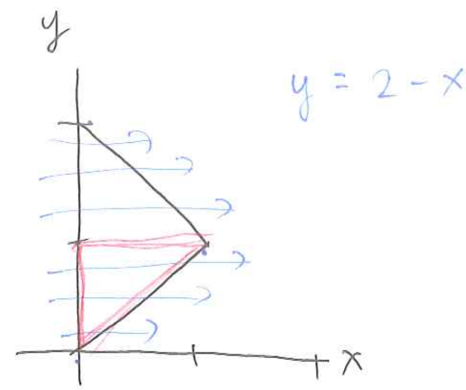
$f(x,y) = \frac{1}{2}xy$

Find $\displaystyle\iint_D \frac{1}{2}xy \, dA$

y first!

$$\int_0^1 \int_x^{2-x} \frac{1}{2} xy \, dy \, dx$$

x first:



$$\int_0^1 \int_0^y \frac{1}{2} xy \, dx \, dy$$

$$+$$

$$\int_1^2 \int_0^{2-y} \frac{1}{2} xy \, dx \, dy$$

We've been looking at $\iint f(x,y) \, dx \, dy$. What is

$\int f(x,y) \, dx$ or $\int f(x,y) \, dy$?

a function of y a function of x