which is the most likely.

) At least 1 six on 6 rolls of a fair die ~10 vot

B) At least 2 sixes on 12 rolls

) At least 3 sixes on 18 rolls. ~30 vote

(at least 1 six on 6 rolls) = 1 − P(no sixes on 6 rolls)

$$= 1 - \underbrace{\left(\frac{5}{6}\right)^6}_{\text{independent events - see later}} \simeq 0.665$$

(B) = 1 − P(no sixes in 12 rolls) − P(exactly 1 six in 12 rolls).

$$= 1 - \left(\frac{5}{6}\right)^{12} - 12 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{11} = 0.619$$

$$(c) = 1 - \sum_{k=0}^{2} P(\text{exactly } k \text{ sixes in 18 rolls})$$

$$= 1 - \sum_{k=0}^{2} \binom{18}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{18-k}.$$

$$= 0.597$$

$P(A) > P(B) > P(c)$.

independence

2 events, one event gives you no information about the other event.

Definition

Events A and B are independent if $P(A \cap B) = P(A) \times P(B)$.

contrast this with disjoint events where both events cannot occur at the same time.

A, B, C are independent if

$$P(A, B) = P(A)\, P(B)$$
$$P(A, C) = P(A)\, P(C)$$
$$P(B, C) = P(B)\, P(C)$$
$$P(A, B, C) = P(A)\, P(B)\, P(C)$$

pairwise independent

often write $P(A, B)$ for $P(A \cap B)$

"probability of A and B"

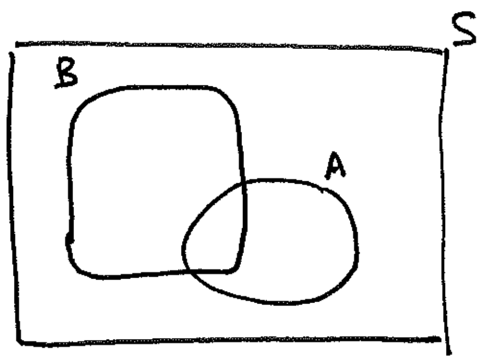
Conditional Probability.

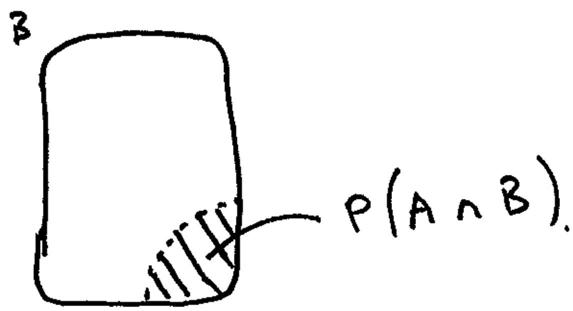
$P(A \mid K_1)$ — conditional probability of event A if event K_1 has already happened.

Definition.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad \text{if } P(B) > 0$$

"probability of A *given* B"





and renormalize so that total probability in B is 1 equency definition.

roll 6 dice.

A : one of the rolls is a 6

B : one of the rolls is a 1

$$P(A|B) = \frac{\#\text{ of times both A and B occur}}{\#\text{ times B occurs.}}$$

1, 1, 2, 4, 6, 2. ← A, B

2, 3, 2, 5, 4, 2

1, ~~4~~, 6, 3, 2, 2. ← A, B.

4, 5, 4, 2, 2, 3

⋮

1, 2, 3, 4, 5, 4 ← B

look over all 6^6 possible outcomes of rolling 6 dice.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A|B)\,P(B)$$
$$= P(B|A)\,P(A)$$

$$\Rightarrow \quad P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

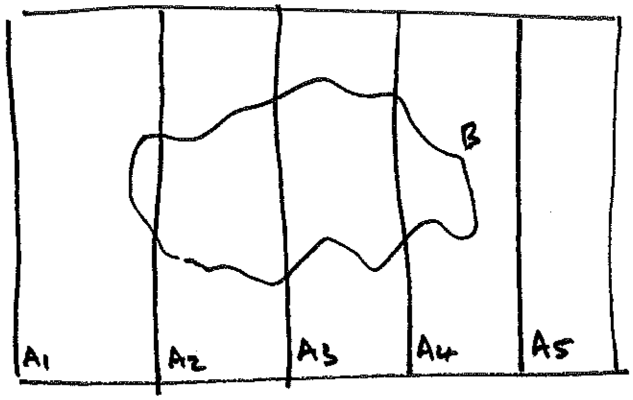
Bayes Theorem.

inductive inference

- how to update what we know about the probability of event A when given information about event B.

$$P(A_1, \ldots, A_n) = P(A_1)\,P(A_2|A_1)\,P(A_3|A_2, A_1) \ldots P(A_n|A_1 \ldots A_{n-1})$$
$$= P(A_n|A_{1:n-1}) \ldots P(A_3|A_2 A_1)\,P(A_2|A_1)\,P(A_1)$$

S



$P(B) = ?$

<u>Partition</u> S into $A_1 \dots A_n$

$$P(B) = P(B \cap A_1) + P(B \cap A_2) + \dots + P(B \cap A_n)$$

all $B \cap A_i$ are disjoint

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \dots P(B|A_n)$$

Law of Total Probability.
(LOTP).

How useful this is depends on
the choice of the partitions $A_1, \dots A_n$.

Disease affects 1% of the population. ("lurgy").

Test is 95% accurate.

patient tests positive.

what's the probability that they have the disease?

D: the specific patient has the disease

T: the specific patient tests positive.

$P(D \mid T)$ — the patient cares about the prob. that they have the disease given a positive test.

$P(T \mid D) = 0.95$

$P(T^c \mid D^c) = 0.95$

$$P(D \mid T) = \frac{P(T \mid D)\, P(D)}{P(T)}$$ Bayes thm.

what is $P(T)$?

$$P(T) = P(T \mid D)\, P(D) + P(T \mid D^c)\, P(D^c)$$

LOTP using partition into people with + without the disease.

$$P(D \mid T) = \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.05 \times 0.99}$$

$$= \underline{\underline{0.16.}}$$

<u>Why?</u> - interplay between how rarely the test is wrong, and how rarely someone has the disease.

~~Frequency~~

Frequency Definition

1000 patients

990 don't have the disease

When we test these 990 people, 5% of them will have a positive test, or ≈ 50 people.

10 have the disease → all 10 test positive.

$$P\left(\begin{array}{l}\text{someone who tests positive} \\ \text{does have the disease}\end{array}\right) = \frac{\#\text{ people with +ve test } \underline{\text{and}}\text{ d}}{\#\text{ positive test}}$$

$$\simeq \frac{10.}{50+10} = \frac{1}{6} \simeq 16\%.$$

Common Conditional Probability Errors.

confusing $P(A \mid B)$ with $P(B \mid A)$ "prosecutors fallacy"

A – person is pregnant

B – person is a woman.

$P(\cancel{A\mid}$

$P(B \mid A) = P(\text{woman} \mid \text{pregnant}) = 1$

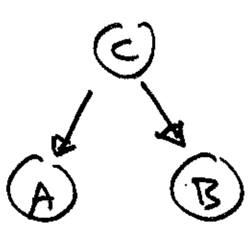
$P(A \mid B) = P(\text{pregnant} \mid \text{woman}) \triangleq \frac{1.5}{80}$

Confusing $P(A)$ "prior" with $P(A \mid B)$ "posterior"

"given that A occurs" __does__ __not__ imply $P(A) = 1$

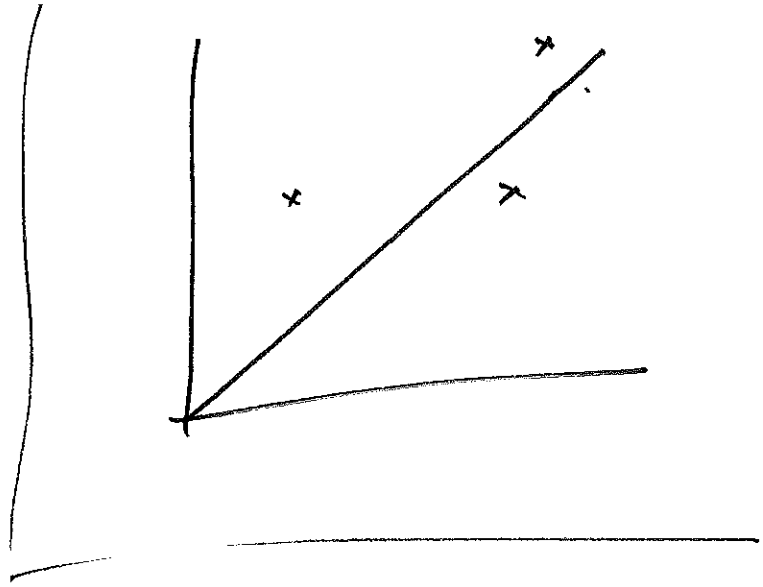
__does__ imply $P(A \mid A) = 1$

confusing independence and conditional independence



efinition.

Events A and B are conditionally independent given C if



$$P(A \wedge B \mid C) = P(A \mid C) \times P(B \mid C)$$

independence $\not\Leftrightarrow$ conditional independence.

series of games of cards against an opponent of unknown strength.

conditional on the strength of the opponent, outcomes of the series of games are independent.

the games are not independent - early games give you information about the outcomes of later games.

(un conditional independence means that earlier games tell you nothing about later games).

...es independence imply conditional independence.

- phenomena with multiple causes.

A - fire alarm goes off

2 possible causes: F - fire

T - burned toast

$P(F \mid A, T^c) = 1$ (assuming no other possible causes of the alarm).

F and T are <u>not</u> conditionally independent given A.