(at least 1 six on 6 rolls) = 1 - \text{p}( \text{no sixes on 6 rolls})

= 1 - \left(\frac{5}{6}\right)^6 = 0.665

\text{independent events}

\text{see later}

(B) = 1 - \text{p}( \text{no sixes in 12 rolls}) - \text{p}( \text{exactly 1 six in 12 rolls})

= 1 - \left(\frac{5}{6}\right)^{12} - 12 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{11} = 0.619

(c) = 1 - \sum_{k=0}^{2} \text{p}( \text{exactly 12 sixes in 18 rolls})

= 1 - \sum_{k=0}^{2} \binom{18}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{18-k}

= 0.597

P(A) > P(B) > P(C).
Two events, one event gives you no information about the other event.

**Definition**

Events A and B are independent if

\[ P(A \cap B) = P(A) \times P(B) \]

Contrast this with disjoint events where both events cannot occur at the same time.

A, B, C are independent if

\[
egin{align*}
P(A, B) &= P(A) \times P(B) \\
P(A, C) &= P(A) \times P(C) \\
P(B, C) &= P(B) \times P(C) \\
P(A, B, C) &= P(A) \times P(B) \times P(C)
\end{align*}
\]

Often write \( P(A, B) \) for \( P(A \cap B) \)

"probability of A and B"
Conditional Probability.

\[ P(A | K_i) \] conditional probability of event A if event \( K_i \) has already happened.

\[ P(A | B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) > 0 \]

"Probability of \( A \) given \( B \)"

\[ P(A \cap B) \]

and renormalize so that \( P(B) \) total probability in \( B \) is 1
Roll 6 dice.

A: one of the rolls is a 6
B: one of the rolls is a 1

\[ P(A|B) = \frac{\text{# of times both A and B occur}}{\text{# times B occurs}}. \]

1, 1, 2, 4, 6, 2. \( \neq \) A, B
2, 3, 2, 5, 4, 2
1, 3, 6, 3, 2, 2. \( \neq \) A, B
4, 5, 4, 2, 2, 3
1, 3, 3, 4, 5, 4 \( \neq \) B

Look over all \( 6^6 \) possible outcomes of rolling 6 dice.
\[
P(A | B) = \frac{P(A \cap B)}{P(B)}
\]
\[
P(A \cap B) = P(A | B) P(B)
\]
\[
= P(B | A) P(A)
\]
\[
\Rightarrow P(A | B) = \frac{P(B | A) P(A)}{P(B)}
\]
\[
P(B | A) = \frac{P(A | B) P(B)}{P(A)}
\]

**Bayes' Theorem.**

**Inductive inference**

- how to update what we know about the probability of event A when given information about event B.

\[
P(A_1, ..., A_n) = P(A_1) P(A_2 | A_1) P(A_3 | A_2, A_1) ... P(A_n | A_1, ..., A_{n-1})
\]
\[
= P(A_n | A_2, ..., A_{n-1}) ... P(A_2 | A_1) P(A_1) P(A_1)
\]
\[ P(B) = \ ? \]

Partition \( S \) into \( A_1, \ldots, A_n \)

\[ P(B) = P(B \cap A_1) + P(B \cap A_2) + \ldots + P(B \cap A_n) \]

all \( B \cap A_i \) are disjoint

\[ P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \ldots + P(B|A_n) \]

Law of Total Probability \( (LTP) \).

How useful this is depends on

the choice of the partitions \( A_1, \ldots, A_n \).
Disease affects 1% of the population. ("lurgy").

Test is 95% accurate.

Patient tests positive.

What's the probability that they have the disease?

\[ D : \text{the specific patient has the disease} \]
\[ T : \text{the specific patient tests positive} \]

\[ P(D|T) \] - the patient cares about the prob. that they have the disease given a positive test.

\[ P(T|D) = 0.95 \]
\[ P(T^c | D^c) = 0.95 \]

\[ P(D|T) = \frac{P(T|D)P(D)}{P(T)} \] Bayes Thm.

What is \( P(T) \)?

\[ P(T) = P(T|D)P(D) + P(T|D^c)P(D^c) \] Law of Total Probability

\( P(D) \) is the probability that a person with the disease tests positive.
\[ P(D \mid T) = \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.05 \times 0.99} = 0.16. \]

Why? - Interplay between how rarely the test is wrong, and how rarely someone has the disease.

Frequency

Frequency Definition

1000 patients
990 don't have the disease

When we test these 990 people, 5% of them will have a positive test, or n = 50 people.

10 have the disease \( \Rightarrow \) all 10 test positive.

\[ P(\text{someone who tests positive}) = \frac{\text{# people with true test and}}{\text{# positive test}} \]

\[ \leq \frac{10}{50 + 10} = \frac{1}{6} \leq 16\%. \]
confusing $P(A|B)$ with $P(B|A)$ "prosecutors fallacy"

A - person is pregnant
B - person is a woman.

$P(B|A) = P(\text{woman} | \text{pregnant}) = 1$
$P(A|B) = P(\text{pregnant} | \text{woman}) = \frac{15}{80}$

Confusing $P(A)$ with $P(A|B)$
"prior" "posterior"

"given that A occurs" does not imply $P(A) = 1$
does imply $P(A|A) = 1$

confusing independence
and conditional independence

\[ \begin{array}{c}
\text{C} \\
\downarrow \\
\text{A} \\
\downarrow \\
\text{B}
\end{array} \]
Events A and B are conditionally independent given C if

\[ P(A \cap B | C) = P(A | C) \cdot P(B | C) \]

independence \( \iff \) conditional independence.

A series of games of cards against an opponent of unknown strength.

Conditional on the strength of the opponent, outcomes of the series of games are independent.

The games are not independent - early games give you information about the outcomes of later games.

(unconditional independence means that earlier games tell you nothing about later games).
so independence imply conditional independence.

- phenomena with multiple causes.

A - fire alarm goes off

2 possible causes: F - fire
T - burned toast

\[ P(F | A, T^c) = 1 \] (assuming no other possible causes of the alarm).

F and T are not conditionally independent given A.