

AMS-131. Sample exam questions

NOTE: these questions should be taken as a guide to the *type* of question that might appear on the exam. Do not expect that the questions on the actual exam will be the same as these ones.

1. For X and Y being i.i.d. positive random variables, show that $E[(X + Y)^2] = 2E[X^2] + 2(E[X])^2$
2. Random variable X has a Negative Binomial distribution with parameters r and p . Recall that the Negative Binomial is the distribution of the number of failures in a sequence of i.i.d. Bernoulli(p) trials before the r th success. By considering X as a sum of simpler random variables, show that the variance of the number of failures before the r th success is given by $\frac{r(1-p)}{p^2}$.
3. If X and Y are i.i.d. standard normal, then the pdf of $W = X^2 + Y^2$ is $f_W(w) = \frac{1}{2}e^{-w/2}$.
 - (a) If $R^2 = W$, find the pdf of R .
 - (b) Find $P(X > 2Y + 3)$ in terms of the standard Normal CDF, $\Phi()$.
4. Let U_1, U_2, \dots, U_{60} be i.i.d. Uniform(0,1), and $X = U_1 + U_2 + \dots + U_{60}$.
 - (a) What is the mean and variance of X ?
 - (b) Give an approximation for $P(X > 17)$.
5. A standard 52 card deck is shuffled. Cards are dealt off the top of the deck one at a time. What is the expected number of cards before the first ace? [Hint: Define an indicator random variable for each of the 48 non-aces, where the indicator is 1 if that card is dealt before the first ace, and zero otherwise. Determine the probability that the indicator takes the value one by considering the permutations of the set of 5 cards consisting of the card and the 4 aces. Then use linearity of expectation.]
6. A fair die is rolled until a 6 appears. If the first 6 is on an even numbered roll, Alice wins. If the first six is on an odd numbered roll, Bob wins. What is the probability that Bob wins?
7. There are 10 deer on campus, 5 male and 5 female. If you trap 6 deer, what's the probability that exactly 3 of them will be male?
8. Suppose that the transition matrix representing the successive wet (rain) and dry (no rain) conditions on any given day is as follows:

	dry	wet
dry	0.7	0.3
wet	0.6	0.4

- (a) If it is raining on a given day, what is the probability that it will be dry on at least one of the next three days?
- (b) Suppose that the probability that it will be dry on a certain Wednesday is 0.2. Determine the probability that it will be raining on the Friday of that week.