11 total

Name:______________________ Section: (day/time) ______________________

AMS131-01 - Quiz 1
Thursday 19th April, 2018.

2 points

1. A 10-card deck consists of 5 red cards and 5 blue cards. Six cards are chosen at random. What's the chance that three are red and three are blue?

3 of 5 red $\rightarrow$ $\dfrac{\binom{5}{3} \times \binom{5}{3}}{\binom{10}{6}}$. $\leftarrow$ 3 of 5 blue

$\leftarrow$ number of 6 card hands.

2. A widget inspector inspects 12 widgets and finds that exactly 3 are defective. Unfortunately, the widgets then get all mixed up and the inspector has to find the 3 defective widgets again by testing widgets one by one.

2 points (a) Find the probability that the inspector will now have to test at least 9 widgets.

3 points (b) Find the probability that the inspector will now have to test at least 10 widgets.

a) complementary event - inspector needs to test at most 8 widgets.

This corresponds to the 3 defective widgets being amongst the first 8, which has prob $\binom{8}{3} / \binom{12}{3}$.

$\Rightarrow$ Prob (test at least 9) $= 1 - \binom{8}{3} / \binom{12}{3}$.

b). complementary event - test at most 9

two ways : all the faulty widgets are in the first 9

none of the faulty widgets are in the first 9.

[TURN OVER]

these are disjoint

$\Rightarrow$ prob $= 1 - \dfrac{\binom{9}{3}}{\binom{12}{3}} - \dfrac{1}{\binom{12}{3}}$. $\simeq 0.614$

4 points

3. A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are 2 marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

Alternative solution

$$P(\text{1st ball is green} \mid \text{green ball is removed})$$

$$= \frac{P(\text{green ball is removed} \mid \text{1st ball is green}) \cdot P(\text{1st ball is green})}{P(\text{green ball is removed})}$$

$$= \frac{1 \times \frac{1}{2}}{P(\text{green ball is removed} \mid \text{1st ball is green}) \times P(\text{1st ball is green}) + P(\text{green ball is removed} \mid \text{1st ball is blue}) \times P(\text{1st ball is blue})}$$

$$= \frac{1 \times \frac{1}{2}}{1 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} = \frac{2}{3}.$$

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   (a) Find the probability that the inspector will now have to test at least 9 widgets.

   (b) Find the probability that the inspector will now have to test at least 10 widgets.

a) complimentary event - inspector needs to test at most 8 widgets

This is the prob that the 3 defective widgets are amongst the first 8, which has prob. $\dfrac{\binom{8}{3}}{\binom{12}{3}}$

$\Rightarrow P(\text{test at least } 9) = 1 - \dfrac{\binom{8}{3}}{\binom{12}{3}}.$

b) complimentary event - test at most 9.

two ways: all 3 faulty are in 1st 9.
none of the faulty are in 1st nine.

[TURN OVER]

These are disjoint

$\Rightarrow \text{prob} = 1 - \dfrac{\binom{9}{3}}{\binom{12}{3}} - \dfrac{1}{\binom{12}{3}} \simeq 0.614$

3. A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are 2 marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

A : event that initial marble is green.

B : event that the removed marble is green

C : event that the remaining marble is green

Required prob. is $P(C|B)$. Condition on A.

$$P(C|B) = P(C|B,A) \cdot P(A|B) + P(C|B,A^c) \cdot P(A^c|B).$$

$$= 1 \times P(A|B) + 0 \times P(A^c|B).$$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} = \frac{1 \times \frac{1}{2}}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

$$= \frac{1/2}{1 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} = \frac{2}{3}$$

$$\Rightarrow P(C|B) = \frac{2}{3}$$