out of 12

Name:______________________________ Section: (day/time) ________________________

The solutions I am submitting are my own work. I understand the University's policy on Academic Integrity and the consequences of submitting work which is not truly my own.

Signature: ____________________________________

AMS131 - Quiz 2

Tuesday 22th May, 2018.

You must show working/explain all answers for full credit.

6 total

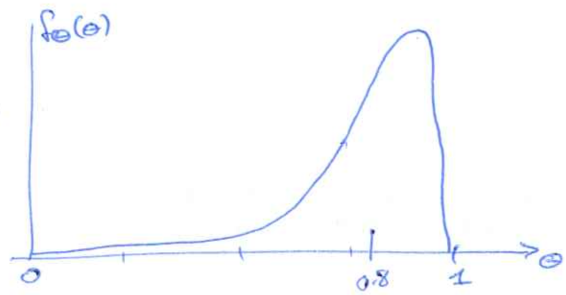
1. A Beta(a,b) distribution is used to represent the pdf $f_\Theta(\theta)$ of the succcess parameter of a series of independent Bernoulli trials. In total, we have observed 8 successes and 2 failures.

   (a) Sketch $f_\Theta(\theta)$ Beta $a=8$ $b=2$.

(1) shape of curve

(1) labelling x-axis (ie cut off at 1)

(1) labelling of y-axis



Mode is at higher value than 0.8.

   (b) What is the probability that the success parameter lies in the range $0.75 \le \theta \le 0.85$?

$$P(0.75 \le \theta \le 0.85) = \int_{0.75}^{0.85} f_x(x)\,dx = \int_{0.75}^{0.85} \frac{\Gamma(10)}{\Gamma(2)\Gamma(8)} x^{8-1}(1-x)^{2-1}\,dx$$

(1) limits of integration

(1) doing the integral

(1) answer.

$$= \frac{9!}{1!\,7!}\int_{0.75}^{0.85} x^7(1-x)\,dx$$

$$= 9\times 8 \times \left[\frac{x^8}{8} - \frac{x^9}{9}\right]_{0.75}^{0.85}$$

$$= 0.29914$$

$$\simeq 0.30$$

[TURN OVER]

6 total

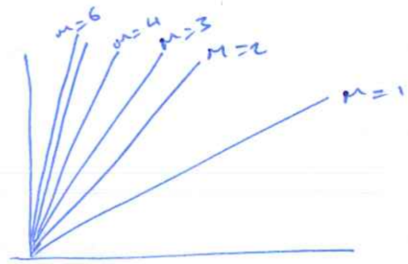
2. When finding the pdf for the slope and intercept of the straight line model $y = mx + c$, I suggested using as the prior distribution on m, $f_M(m) \sim \text{Unif}()$ over some range.

   (a) If $f_M(m) \sim \text{Unif}(0, 1000)$ does this put equal probability mass on lines that are roughly horizontal and lines that are roughly vertical? Explain your answer.

(1) answer

(1) explanation

No, it puts much more prob. mass on lines that are ~ vertical



Most of the lines are near vertical.

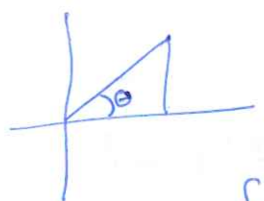
   (b) An alternative prior distriution would be to say that θ, the *angle* between the line and the positive x-axis, has the distribution $\theta \sim \text{Unif}(-\pi/2, \pi/2)$. What distribution does this imply for the slope, m?

(1) relationship bet m, θ

(1) formula for transformation of variables

(1) PDF

(1) stating valid range for m.



$m = \tan\theta$

Transformation of variables

$$f_M(m) = f_\Theta(\theta)\left|\frac{d\theta}{dm}\right| = \frac{f_\Theta(\theta)}{\left|\frac{dm}{d\theta}\right|}$$

$\theta = \tan^{-1} m$

$$\frac{d\theta}{dm} = \frac{1}{1+m^2}$$

$$f_M(m) = \frac{1}{\pi}\,\frac{1}{1+m^2} \qquad -\infty \le m \le \infty$$