AMS131 - Quiz 2
Tuesday 22nd May, 2018.
You must show working/explain all answers for full credit.

1. A Beta(a,b) distribution is used to represent the pdf $f_\theta(\theta)$ of the success parameter of a series of independent Bernoulli trials. In total, we have observed 8 successes and 2 failures.

(a) Sketch $f_\theta(\theta)$

(b) What is the probability that the success parameter lies in the range $0.75 \leq \theta \leq 0.85$?

\[
P(0.75 \leq \theta \leq 0.85) = \int_{0.75}^{0.85} f_\theta(x) \, dx = \int_{0.75}^{0.85} \frac{B(10\theta)}{B(8\theta) B(2(1-\theta))} \theta^{8-1} (1-\theta)^{2-1} \, dx.
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2. When finding the pdf for the slope and intercept of the straight line model \( y = mx + c \), I suggested using as the prior distribution on \( m \), \( f_M(m) \sim \text{Unif}() \) over some range.

(a) If \( f_M(m) \sim \text{Unif}(0, 1000) \) does this put equal probability mass on lines that are roughly horizontal and lines that are roughly vertical? Explain your answer.

(b) An alternative prior distribution would be to say that \( \theta \), the angle between the line and the positive x-axis, has the distribution \( \theta \sim \text{Unif}(-\pi/2, \pi/2) \). What distribution does this imply for the slope, \( m \)?