$\frac{13}{\cancel{12}}$ total

Name:____________________ Section: (day/time) ____________________

AMS131 - Quiz 3
Thursday 31th May, 2018.
You must show working/explain all answers for full credit.

Some (possibly) useful results:
$\text{Cov}(X,Y) = E[(X - E[X])(Y - E[Y])]$
$\text{Var}(aX) = a^2\text{Var}(X)$
$\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X,Y)$

3 points

1. Suppose that X, Y, and Z are three random variables such that $\text{Var}(X) = 1$, $\text{Var}(Y) = 4$, $\text{Var}(Z) = 8$, $\text{Cov}(X,Y) = 1$, $\text{Cov}(X,Z) = -1$, and $\text{Cov}(Y,Z) = 2$. What is $\text{Var}(3X - Y - 2Z + 1)$?

$\text{Var}(3X - Y - 2Z + 1) = 9\text{Var}(X) + \text{Var}(Y) + 4\text{Var}(Z)$ (1)

$+ 2\text{Cov}(3X, -Y) + 2\text{Cov}(3X, -2Z)$

$+ 2\text{Cov}(-Y, -2Z)$

$= 9\text{Var}(X) + \text{Var}(Y) + 4\text{Var}(Z)$ (1)

$-6\text{Cov}(X,Y) - 12\text{Cov}(X,Z)$

$+ 4\text{Cov}(Y,Z)$

$= 9 + 4 + 32$ (1).

$-6 + 12 + 8$

$= 59$

[TURN OVER]

5 points

2. A fair die is rolled, and then a coin with probability p of Heads is flipped as many times as the die roll says, e.g., if the result of the die roll is a 3, then the coin is flipped 3 times. Let X be the result of the die roll and Y be the number of times the coin lands Heads. Find the joint PMF of X and Y. Are they independent?

(1) $P(X,Y) = P(Y|X)\,P(X)$.

(1) $P(Y|X)$ = prob of y successes in x trials

$= \binom{x}{y} p^y (1-p)^{x-y}$

(1) $P(X) = \frac{1}{6}$

(1) $P(X,Y) = \frac{1}{6}\binom{x}{y} p^y (1-p)^{x-y}$ for $x = 1, 2, \ldots 6$, $y = 0, 1 \ldots x$.

(1) They are not independent

$P(Y|X)$ depends on x

~~4~~ 5 points

3. Random variable X has the uniform distribution on the interval $[-2, 2]$ and $Y = X^6$. Show that X and Y are uncorrelated.

(1) $\mathrm{Cov}(X,Y) = E[XY] - E[X]E[Y]$

(1) $E[X] = 0$ by symmetry.

(1) + (1) $E[XY] = E[X^7]$ X^7 is an odd function $\Rightarrow E[X^7] = 0$ as the range is symmetric.

Hence

$\mathrm{Cov}(X,Y) = 0 - 0\,E[Y] = 0$

(1) and $\rho(X,Y) = 0$ i.e. uncorrelated.