Supplementary (Pearson) Problems

1.12 p53 Intro / Counting

Q3, 4, 6, 9

2.5 p90. Conditional Prob / Bayes Thm.

Q 3, 5, 13, 16, 20, 23, 28. (36).

3.11 p202 RVs + Distributions. Q10 when done transformations,

1, 4, 9 ← need more here.

4.9 p272 Expectation.

→ 4.9 4,

4.1 ~~4~~ ~~4~~, 7, 11 ← more needed.

4.2 2, 3, 6, 9 4.3 5, 6, 9 variance of sum.

5.11 p345 Special Distributions

5 (Poisson).

[9 (sum of 2 Poissons is Poisson).]

11 (Geometric)

12

13

18

20

1-12

Q3. $\dfrac{\binom{250}{18}\binom{100}{12}}{\binom{350}{30}}$

Q4 $\binom{20}{10}$ ways of chosing 10 cards.

$\binom{4}{2}$ ways of chosing each of 1, 2, 3, 4, 5 $\Rightarrow \dfrac{\binom{4}{2}^5}{\binom{20}{10}}$

Q6 a) $\binom{r+w}{r}$ ways of placing red balls.

one way that they're on 1st r positions $= \dfrac{1}{\binom{r+w}{r}}$.

b). $\binom{r+1}{r} \Big/ \binom{r+w}{r} = \dfrac{r+1}{\binom{r+w}{r}}$.

\# ways the red balls can be in 1st $r+1$ slots.

Q8. $\binom{10}{7}$ ways of chosing envelopes for the red cards

$\binom{7}{j}\binom{3}{7-j}$ ways of chosing exactly j red envelopes and $7-j$ green envelopes.

$\Rightarrow P(j$ red envelopes contain red cards$) = \dfrac{\binom{7}{j}\binom{3}{7-j}}{\binom{10}{7}}$.

However, if j red envelopes contain red cards then $j-4$ green envelopes contain green cards.

$\Rightarrow$ this is the prob of $k = j + (j-4) = 2j-4$ matches.

2.5. Q3 $P(A^c \cup B^c)$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A\cap B)}{P(B)} \qquad \frac{P(A\cap B)}{1/5} + \frac{P(A\cap B)}{1/3} = \frac{2}{3}.$$

$$\Rightarrow P(A\cap B) = 1/12.$$

$$P(A^c \cup B^c) = 1 - \frac{1}{12} = \frac{11}{12}.$$

Q5 $P(1^{st}\ 3 \text{ are } 6 \mid \text{exactly } 3 \text{ out of } 10 \text{ are } 6)$

$$= \frac{\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{10-3}}{\binom{10}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{10-3}} = \frac{1}{\binom{10}{3}}.$$

Q13

a) $P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$

$= 0.3 + 0.5 + 0.8 - [0.3\times 0.5 + 0.3\times 0.8 + 0.5\times 0.8] + 0.3\times 0.5\times 0.8$

$= 0.93$

b) $P(A\cap B^c\cap C^c) + P(A^c\cap B^c\cap C^c) + P(A^c\cap B^c\cap C) = 0.38.$

Q16 : 5 balls $\rightarrow$ n boxes.

P(no box contains >2 balls)

A_i - event that box i has at least 3 balls.

$$P(A_i) = \sum_{j=3}^{5} P(\text{Box } i \text{ has exactly } j \text{ balls}).$$

$$= \binom{5}{3}\left(\frac{n-1}{n}\right)^2\left(\frac{1}{n}\right)^3 + \binom{5}{4}\left(\frac{1}{n}\right)^4\frac{n-1}{n} + \frac{1}{n^5} = p$$

A_i's are disjoint $\Rightarrow$ P(at least one of A_i's) $= np$.

$\Rightarrow$ P(no box contains >2 balls) $= 1-np$.

Q20 E - event that B wins.

1st turn: B wins if A misses + b wins, prob $\frac{5}{6} \times \frac{1}{6}$.

or. both miss on 1st turn, + subsequently B wins.

$$P(E) = \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} P(E) \qquad P(E) = \frac{5}{11}$$

or. sum series $\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \ldots$

Q23.

A - meet statistician, B - person is shy.

~~B - meet econ~~

$$P(A \mid B) = \frac{0.8 \times 0.1}{0.8 \times 0.1 + 0.15 \times 0.9} = 0.372.$$

Q28. a)

$$P(X = n-1 \mid X \geq n-2) = \frac{P(X = n-1)}{P(X \geq n-2)}$$

$$= \frac{\binom{n}{n-1}\left(\frac{1}{2}\right)^n}{\left[\binom{n}{n-2} + \binom{n}{n-1} + \binom{n}{n}\right]\left(\frac{1}{2}\right)^n}$$

$$= \frac{n}{\frac{n(n-1)}{2} + n + 1} = \frac{2n}{n^2 + n + 2}.$$

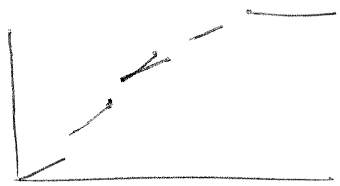
b). $= P(\text{exactly 1 head in last 2 tosses}) = \frac{1}{2}.$

④

3.11 Q1 $F(z) = P(Z \le z)$

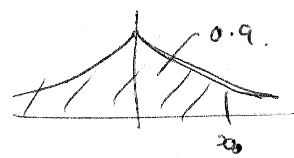
$= P(Z = X) P_r(X \le z) + P(Z = Y) P(Y \le z)$

$= \frac{1}{2} P_r(X \le z) + \frac{1}{2} P(Y \le z)$



(approximately).

Q4. $f(x) = \frac{1}{2} e^{-|x|} \qquad -\infty \le x \le \infty.$



area to left of $x = 0$ $= 1/2$ by symmetry

$\Rightarrow$ require $\displaystyle\int_0^{x_0} \frac{1}{2} e^{-x}\, dx = 0.4$

$\left[-\frac{1}{2} e^{-x}\right]_0^{x_0} = \frac{1}{2}\left(1 - e^{-x_0}\right)$

$\frac{1}{2}\left(1 - e^{-x_0}\right) = 0.4$

$1 - e^{-x_0} = 0.8.$

$0.2 = e^{-x_0}$

$\log(0.2) = -x_0 \qquad x_0 = \log(5)$

Q9. A . tack lands point up on all 3 tosses. $\leftarrow$ indicator RV.

$P(A | X = x) = x^3$

$P_r(A) = \displaystyle\int_0^1 x^3 f(x)\, dx = \frac{1}{10}$ $\leftarrow$ $P(A) = E[$ indicator RV for A $]$

4.1 Q7.

$E\left(\frac{1}{x}\right) = \int_0^1 \frac{1}{x}\,dx = \left[\log x\right]_0^1$ — infinite $\Rightarrow$ expectation does not exist.

4.3 Q9 mean $\frac{n+1}{2}$

$$E[X^2] = \sum x^2 P(X = x)$$

$$= \frac{1}{n}\sum_{j=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{n \cdot 6} = \frac{(n+1)(2n+1)}{6}.$$

$$\text{var}(X) = \cancel{n+1 - \frac{n+1}{2} = \frac{n+1}{2}}.$$

$$\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = \frac{n^2-1}{12}.$$

4.9 Q4

$$E(X+Y+Z) = E(X) + E(Y) + E(Z)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}.$$ — if marginals are uniform

but, since $X+Y+Z < 1.3$ this is impossible.

5.11 Q5 $\bar{X} < \frac{1}{2}$ implies all 4 observations are zero ①

or 1 is 1 and 3 are zero. ②

① has prob $(\exp(-\lambda))^4$

② $4 \times (\exp(-\lambda))^3 \times \lambda \exp(-\lambda)$

total prob $(4\lambda + 1)(\exp(-\lambda))^4$

$$= (4\lambda + 1)\, e^{-4\lambda}$$

5.11 Q11

$P(\text{at least 1 is successful}) = \frac{1}{3} + \frac{1}{5} - \frac{1}{3} \times \frac{1}{5} = \frac{7}{15}$.

days to successful launch ~ $FS\left(\frac{7}{15}\right)$

$E\{\text{\# days}) = \frac{15}{7}$

Q12 $X > n$ iff 1st n are either all H or all T

$P(X > n) = \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{n-1}$

For $n = 2, 3 \ldots$

$$P(X = n) = P(X > n-1) - P(X > n)$$
$$= \left(\frac{1}{2}\right)^{n-2} - \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^{n-1}$$

Q13. $\lambda = 120 \times \frac{1}{36} = \frac{10}{3}$

$$P(X=3) = \frac{\exp\left(-\frac{10}{3}\right)\left(\frac{10}{3}\right)^3}{3!} = 0.2202.$$

Q18. Sample size is small wrt population

$\Rightarrow$ dist. is Binomial $\left(n = 200,\ p = \frac{15\,000}{500{,}000}\right)$.

$\lambda = np = 6$

$P(X < 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$.

$= 0.0025 + 0.0149 + 0.446 + 0.892 = 0.1512$.

Q20. $X \sim \text{Binom}(n, p)$.

$Y \sim$ Negative Binom (r, p) r pos. integer.

$$\underbrace{P(X < r)}_{\text{less than } r \text{ successes in } n \text{ Bernoulli trials}} = \underbrace{P(Y > n-r)}_{\text{more than } n-r \text{ failures before } r \text{ successes}}$$

ie more than n trials needed to get r successes.